

# Sémantique des Langages de Programmation (SemLP)

## TD n° 1 : Operational Semantics

### Exercice 1 : Implementing Imp (Ex. 4 in the course notes)

Implement in your favorite programming language the big-step and small-step reduction rules of the Imp language given in Figure 1 and Figure 2.

$$\begin{array}{c}
 \frac{}{(v, s) \Downarrow v} \quad \frac{}{(x, s) \Downarrow s(x)} \quad \frac{(e, s) \Downarrow v \quad (e', s) \Downarrow v'}{(e + e', s) \Downarrow (v +_{\mathbf{Z}} v')} \quad \frac{(e, s) \Downarrow v \quad (e', s) \Downarrow v'}{(e < e', s) \Downarrow (v <_{\mathbf{Z}} v')} \\
 \\
 \frac{}{(\text{skip}, s) \Downarrow s} \quad \frac{(e, s) \Downarrow v}{(x := e, s) \Downarrow s[v/x]} \quad \frac{(S_1, s) \Downarrow s' \quad (S_2, s') \Downarrow s''}{(S_1; S_2, s) \Downarrow s''} \\
 \\
 \frac{(b, s) \Downarrow \text{true} \quad (S, s) \Downarrow s'}{(\text{if } b \text{ then } S \text{ else } S', s) \Downarrow s'} \quad \frac{(b, s) \Downarrow \text{false} \quad (S', s) \Downarrow s'}{(\text{if } b \text{ then } S \text{ else } S', s) \Downarrow s'} \\
 \\
 \frac{(b, s) \Downarrow \text{false}}{(\text{while } b \text{ do } S, s) \Downarrow s} \quad \frac{(b, s) \Downarrow \text{true} \quad (S; \text{while } b \text{ do } S, s) \Downarrow s'}{(\text{while } b \text{ do } S, s) \Downarrow s'} \quad \frac{(S, s) \Downarrow s'}{(\text{prog } S, s) \Downarrow s'}
 \end{array}$$

FIGURE 1 – Imp big-step reduction

$$\begin{array}{l}
 (x := e, K, s) \quad \rightarrow \quad (\text{skip}, K, s[v/x]) \quad \text{if } (e, s) \Downarrow v \\
 (S; S', K, s) \quad \rightarrow \quad (S, S' \cdot K, s) \\
 (\text{if } b \text{ then } S \text{ else } S', K, s) \quad \rightarrow \quad \begin{cases} (S, K, s) & \text{if } (b, s) \Downarrow \text{true} \\ (S', K, s) & \text{if } (b, s) \Downarrow \text{false} \end{cases} \\
 (\text{while } b \text{ do } S, K, s) \quad \rightarrow \quad \begin{cases} (S, (\text{while } b \text{ do } S) \cdot K, s) & \text{if } (b, s) \Downarrow \text{true} \\ (\text{skip}, K, s) & \text{if } (b, s) \Downarrow \text{false} \end{cases} \\
 (\text{skip}, S \cdot K, s) \quad \rightarrow \quad (S, K, s)
 \end{array}$$

FIGURE 2 – Imp small-step reduction

### Exercice 2 : Break and Continue (Ex. 5 in the course notes)

Extend the Imp language with the commands **break** and **continue** by providing their big-step and small-step reduction rules. Their informal semantics is as follows :

**break** causes the execution of the nearest enclosing **while** statement to be terminated.

Program control is immediately transferred to the point just beyond the terminated statement. It is an error for a **break** statement to appear where there is no enclosing **while** statement.

`continue` causes the execution of the nearest enclosing `while` statement to be terminated. Program control is immediately transferred to the end of the body, and the execution of the affected `while` statement continues from that point with a reevaluation of the loop test. It is an error for `continue` to appear where there is no enclosing `while` statement.

*Hint* : for the big-step, consider extended judgments of the shape  $(S, s) \Downarrow (o, s')$  where  $o$  is an additional information indicating the mode of the result, for the small-step consider a new continuation  $\text{endloop}(K)$ , where  $K$  is an arbitrary continuation.

### Exercice 3 : Hoare-Floyd Assignment Rule (Ex. 8 in the course notes)

Suppose that  $A$  is a first-order formula. Show the validity of the triple

$$\{A[e/x]\} x := e \{A\}$$

Moreover, show that the alternative triple

$$\{A\} x := e \{A[e/x]\}$$

is *not* valid.

### Exercice 4 : Hoare-Floyd Rules

Show the validity of each of the Hoare-Floyd rules given in Figure 3.

$$\frac{A \subseteq A' \quad \{A'\} S \{B'\} \quad B' \subseteq B}{\{A\} S \{B\}} \quad \frac{\{A\} S_1 \{C\} \quad \{C\} S_2 \{B\}}{\{A\} S_1; S_2 \{B\}}$$

$$\frac{\{A \cap b\} S_1 \{B\} \quad \{A \cap \neg b\} S_2 \{B\}}{\{A\} \text{if } b \text{ then } S_1 \text{ else } S_2 \{B\}} \quad \frac{A \subseteq B}{\{A\} \text{skip} \{B\}}$$

$$\frac{A; R_{x:=e} \subseteq B}{\{A\} x := e \{B\}} \quad \frac{(A \cap \neg b) \subseteq B \quad \{A \cap b\} S \{B\}}{\{A\} \text{while } b \text{ do } S \{B\}}$$

FIGURE 3 – Floyd-Hoare rules for `Imp`