Sémantique des Langages de Programmation (SemLP)
TD n° 3 : Unification

Exercice 1 : (Ex. 55 in the course notes)

Show that if S is a substitution unifying the system $\{s_1 = s_2, x = t\}$ then S unifies $\{s_1[t/x] = s_2[t/x]\}$ as well.

Exercice 2 : (Ex. 57 in the course notes)

Apply the unification algorithm viewed in the course to the following systems of equations :

1. $\{\ f(x, f(x, y)) = f(g(y), f(g(a), z))\ \}$

2. $\{\ f(x, g(y)) = f(y, g(g(x)))\ \}$

Exercice 3 : * (Ex. 61 in the course notes)**

1. Propose a method to transform a unification problem of the shape :

$$E = \{\ t_1 = s_1,\ \ldots,\ t_n = s_n\ \}$$

over the signature $\Sigma = \{g_1, ..., g_m\}$ with $n, m \geqslant 1$ into a unification problem E' with the following properties :

1. E' contains exactly one equation,
2. the terms in E' range over the signature $\Sigma' = \{f\}$, where f is binary,
3. E has a solution if and only if E' has a solution, and

2. Apply the method to the system below, where x, y and z are variables.

$$E = \{\ x = h(y),\ g(c, x, y) = g(y, z, z)\ \}$$

Exercice 4 : (Ex. 62 in the course notes)

Let $t, s, \ldots$ be terms over the signature Σ. We say that t is a *filter* for s if there exists a substitution S with $S\,t = s$. We denote this fact as $t \leqslant s$ [1]. Prove or disprove the following assertions :

1. If $t \leqslant s$, then t and s are unifiable.

2. If t and s are unifiable, then $t \leqslant s$ and $s \leqslant t$.

3. it $t \leqslant s$ and $s \leqslant t$, then s and t are unifiable.

4. For all t, s there exists an r with $r \leqslant t$, and $r \leqslant s$.

5. For all t, s, there exists an r with $t \leqslant r$ and $s \leqslant r$.

1. Not to be confused with the notation $S \leqslant S'$ over substitutions.