Sémantique des Langages de Programmation (SemLP)
TD n° 1 : Operational Semantics

Exercice 1 : Implementing Imp (Ex. 4 in the course notes)

Implement in your favorite programming language the big-step and small-step reduction rules of the Imp language given in Figure 1 and Figure 2.

$$\frac{}{(v, s) \Downarrow v} \qquad \frac{}{(x, s) \Downarrow s(x)} \qquad \frac{(e, s) \Downarrow v \qquad (e', s) \Downarrow v'}{(e + e', s) \Downarrow (v +_{\mathbf{Z}} v')} \qquad \frac{(e, s) \Downarrow v \qquad (e', s) \Downarrow v'}{(e < e', s) \Downarrow (v <_{\mathbf{Z}} v')}$$

$$\frac{}{(\mathsf{skip}, s) \Downarrow s} \qquad \frac{(e, s) \Downarrow v}{(x := e, s) \Downarrow s[v/x]} \qquad \frac{(S_1, s) \Downarrow s' \qquad (S_2, s') \Downarrow s''}{(S_1; S_2, s) \Downarrow s''}$$

$$\frac{(b, s) \Downarrow \mathsf{true} \qquad (S, s) \Downarrow s'}{(\mathsf{if}\ b\ \mathsf{then}\ S\ \mathsf{else}\ S', s) \Downarrow s'} \qquad \frac{(b, s) \Downarrow \mathsf{false} \qquad (S', s) \Downarrow s'}{(\mathsf{if}\ b\ \mathsf{then}\ S\ \mathsf{else}\ S', s) \Downarrow s'}$$

$$\frac{(b, s) \Downarrow \mathsf{false}}{(\mathsf{while}\ b\ \mathsf{do}\ S, s) \Downarrow s} \qquad \frac{(b, s) \Downarrow \mathsf{true} \qquad (S; \mathsf{while}\ b\ \mathsf{do}\ S, s) \Downarrow s'}{(\mathsf{while}\ b\ \mathsf{do}\ S, s) \Downarrow s} \qquad \frac{(S, s) \Downarrow s'}{(\mathsf{prog}\ S, s) \Downarrow s'}$$

FIGURE 1 – Imp big-step reduction

$$\begin{array}{lcl}
(x := e, K, s) & \to & (\mathsf{skip}, K, s[v/x]) \quad \text{if } (e, s) \Downarrow v \\
(S; S', K, s) & \to & (S, S' \cdot K, s) \\
(\mathsf{if}\ b\ \mathsf{then}\ S\ \mathsf{else}\ S', K, s) & \to & \begin{cases} (S, K, s) & \text{if } (b, s) \Downarrow \mathsf{true} \\ (S', K, s) & \text{if } (b, s) \Downarrow \mathsf{false} \end{cases} \\
(\mathsf{while}\ b\ \mathsf{do}\ S, K, s) & \to & \begin{cases} (S, (\mathsf{while}\ b\ \mathsf{do}\ S) \cdot K, s) & \text{if } (b, s) \Downarrow \mathsf{true} \\ (\mathsf{skip}, K, s) & \text{if } (b, s) \Downarrow \mathsf{false} \end{cases} \\
(\mathsf{skip}, S \cdot K, s) & \to & (S, K, s)
\end{array}$$

FIGURE 2 – Imp small-step reduction

Exercice 2 : Break and Continue (Ex. 5 in the course notes)

Extend the Imp language with the commands break and continue by providing their big-step and small-step reduction rules. Their informal semantics is as follows :

break causes the execution of the nearest enclosing while statement to be terminated. Program control is immediately transferred to the point just beyond the terminated statement. It is an error for a break statement to appear where there is no enclosing while statement.

continue causes the execution of the nearest enclosing while statement to be terminated. Program control is immediately transferred to the end of the body, and the execution of the affected while statement continues from that point with a reevaluation of the loop test. It is an error for continue to appear where there is no enclosing while statement.

Hint : for the big-step, consider extended judgments of the shape $(S, s) \Downarrow (o, s')$ where o is an additional information indicating the mode of the result, for the small-step consider a new continuation $\mathsf{endloop}(K)$, where K is an arbitrary continuation.

Exercice 3 : Hoare-Floyd Assignment Rule (Ex. 8 in the course notes)

Suppose that A is a first-order formula. Show the validity of the triple

$$\{A[e/x]\}\ x := e\ \{A\}$$

Moreover, show that the alternative triple

$$\{A\}\ x := e\ \{A[e/x]\}$$

is *not* valid.

Exercice 4 : Hoare-Floyd Rules

Show the validity of each of the Hoare-Floyd rules given in Figure 3.

$$\frac{A \subseteq A' \qquad \{A'\}\ S\ \{B'\} \qquad B' \subseteq B}{\{A\}\ S\ \{B\}} \qquad \frac{\{A\}\ S_1\ \{C\} \qquad \{C\}\ S_2\ \{B\}}{\{A\}\ S_1; S_2\ \{B\}}$$

$$\frac{\{A \cap b\}\ S_1\ \{B\} \qquad \{A \cap \neg b\}\ S_2\ \{B\}}{\{A\}\ \mathsf{if}\ b\ \mathsf{then}\ S_1\ \mathsf{else}\ S_2\ \{B\}} \qquad \frac{A \subseteq B}{\{A\}\ \mathsf{skip}\ \{B\}}$$

$$\frac{A; R_{x:=e} \subseteq B}{\{A\}\ x := e\ \{B\}} \qquad \frac{(A \cap \neg b) \subseteq B \qquad \{A \cap b\}\ S\ \{B\}}{\{A\}\ \mathsf{while}\ b\ \mathsf{do}\ S\ \{B\}}$$

FIGURE 3 – Floyd-Hoare rules for Imp